COSC581 Homework 1

Due Thursday, January 30, 2025.

1. Prove that ( f = 𝛰(g) and f = Ω(g) ) ⇔ f = Θ(g). (This can absolutely be a direct proof, allow yourself to use common sense and definitions. Just practice writing formal and concise work)
2. Show x < 2x for x ≥ 1 using induction.
3. If r is a real number such that r2 = 2, then r is an irrational number. (prove via contradiction)
4. For the following set of functions, tell me which f/g pairs demonstrate which of the 6 asymptotic behaviors we discussed in class today. SHOW YOUR WORK.
   1. f = n3 + 4, g = n3 + 8n + log(n)
   2. f = 2n, g = n
   3. f = log(2n), g = n
   4. f = nlog(n), g = nn
   5. f = logn^(log17), g = log17^(logn)
5. Using the formal definition, justify why f can NOT be o(g) and Ω(g).
6. A little fun with pseudo-randomness\*. Suppose we start with an edgeless graph of order, say, 100. Then we begin uniformly generating edges (pairs of integers between 1 and 100) without replacement, stopping as soon as we produce in our graph a P5, a C5, or a K5 subgraph. Which of these three events is most likely and why?

\**If this question seems too complex, turn your brain off. You’re thinking too hard. (That’s the magic of this question) Don’t ask ChatGPT.*